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Modelling Human Population Dynamics with Delay Differential Equations and Machine Learning

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ABSTRACT

This paper explores a mathematical model of human population dynamics using Delay Differential Equations (DDEs), integrating factors like birth rates, mortality rates, migration, and demographic shifts. It details the construction and simulation of the model, which accounts for temporal delays in population responses. Results reveal insights into population trends under various scenarios, emphasizing the impact of policy, environmental changes, and socio-economic factors. The chapter also covers the dynamical systems theory underlying the model and outlines a systematic approach for data modeling, including verification, analysis, and machine learning techniques.

Keywords: - Human population dynamics, Delay Differential Equations, simulation, policy impact, data modelling.

1. Introduction

1.1 Introduction to Modeling Human Population Dynamics Using Delay Differential Equations

Understanding and predicting human population dynamics is a critical endeavor with profound implications for various fields, including ecology, economics, public health, and urban planning. To gain insights into the complex processes governing population growth, migration, and interactions, mathematical models are indispensable tools. Human population dynamics are influenced by a myriad of factors, including birth and death rates, immigration and emigration, environmental changes, and resource availability. These factors often exhibit time delays in their effects, making traditional ordinary differential equations (ODEs) insufficient for capturing the nuances of population behavior over time. Delay Differential Equations (DDEs), on the other hand, allow us to model how past events influence future population dynamics, providing a more accurate representation of real-world phenomena [1-6].

In this exploration of human population dynamics using DDEs, we will delve into the key components of such a mathematical model. We will consider birth and death rates, immigration and emigration, and the concept of carrying capacity as essential elements. The model aims to describe how these factors interact and how the population size changes over time while accounting for the time delays involved in various processes. While the presented model is a simplified representation of human population dynamics, it serves as a foundational framework for more complex and realistic models. The integration of time delays allows us to explore questions related to population stability, growth, and fluctuations in a dynamic world where events from the past continue to shape the future [6-8].

In the subsequent sections, we will investigate the mathematical formulation of the DDE model, discuss parameter estimation, and explore numerical methods for solving and analyzing population dynamics. We will also emphasize that this model is just a starting point and can be extended to incorporate additional complexities, such as age structure, disease spread, and spatial dynamics, to provide a more comprehensive understanding of the intricate dynamics of human populations.

Modeling human population dynamics using DDEs is a complex task, but it can provide valuable insights into how populations change over time, especially when there are time delays involved in various processes. Below, we outline a basic mathematical model using DDEs to describe human population dynamics. Please note that this is a simplified model, and real-world population dynamics involve many more factors and complexities. We consider a simple population model with the following components:
1. Birth Rate (B): The rate at which new individuals are born into the population.
2. Death Rate (D): The rate at which individuals die from the population.
3. Immigration Rate (I): The rate at which individuals from outside the population move in.
4. Emigration Rate (E): The rate at which individuals from the population move out.
5. Carrying Capacity (K): The maximum population size that the environment can sustain.

Now, let's construct a delay differential equation (DDE) that incorporates these factors. Let $N(t)$ be the population size at time t. The rate of change in population size, $\frac{dN}{dt}$, can be modeled as follows:

$$\frac{dN}{dt} = B(N(t), I(t - \tau)) - D(N(t), E(t - \tau)) - \frac{N(t)}{K}$$

In this equation:
- The first term represents the birth rate, which depends on the current population $N(t)$ and the delayed effect of immigration $I(t - \tau)$ (assuming a time delay τ for immigration to take effect).
- The second term represents the death rate, which depends on the current population $N(t)$ and the delayed effect of emigration $E(t - \tau)$.
- The third term represents the carrying capacity K, which limits population growth [9-11].

This model assumes that immigration and emigration have time delays, which can be important when considering how long it takes for individuals to migrate and their impact on the population. Parameters like birth rates, death rates, immigration rates, emigration rates, and carrying capacity should be estimated based on empirical data or other relevant information. Also, you can introduce additional complexity by considering age structure, disease dynamics, and other factors that influence population dynamics. Solving this DDE numerically would provide a time-dependent population trajectory that accounts for time delays in immigration and emigration. You can use various numerical methods, such as Runge-Kutta methods or MATLAB's built-in DDE solver, to simulate the population dynamics and analyze the results. Keep in mind that real-world population dynamics are much more intricate, and this simplified model serves as a starting point. More advanced models can incorporate spatial dynamics, age structure, disease spread, and other factors for a more realistic representation of human population dynamics [12].

1.2 Population Dynamics Significance

The significance of studying population dynamics lies in its profound implications for a wide range of fields and the understanding of how human societies interact with the environment. Here are some key points highlighting the significance of population dynamics:

Resource Allocation and Planning

Population dynamics data is crucial for governments and policymakers to allocate resources effectively. It helps in planning for public services such as healthcare, education, and infrastructure to meet the needs of a growing or shrinking population. This data serves as the foundation for informed decisions regarding the allocation of resources for essential public services. By understanding how a population is growing or shrinking, policymakers can tailor their strategies to meet the evolving needs of their communities. Whether it's expanding healthcare facilities to accommodate a growing population or optimizing educational programs to match changing student demographics, population dynamics data empowers governments to make well-informed, proactive decisions that enhance the quality of life for their citizens [13-15].

2. MATHEMATICAL MODEL ON HUMAN POPULATION DYNAMICS

Human population dynamics is a complex field of study that involves understanding how populations grow and change over time. One of the mathematical tools used to model such dynamics is Delay Differential Equations (DDEs), which consider the fact that the rate of change in population can depend on past states of the population.

2.1 Introduction to Delay Differential Equations (DDEs)

A Delay Differential Equation is a type of differential equation where the derivative of the function at a certain time is given in terms of the function's values at previous times. In the context of population dynamics, DDEs are useful for modeling scenarios where there is a time lag between changes in population size and their effects on the birth and death rates.

2.2 Basic Model Without Delay

Consider a simple population growth model described by the ordinary differential equation (ODE):

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right)$$

where:

- P(t) is the population size at time t.
- r is the intrinsic growth rate.
- K is the carrying capacity of the environment.

2.3 Incorporating Delay

To incorporate delay, we consider that the growth rate at time ttt depends not only on the current population size but also on the population size at some previous time $t-\tau$ where τ is the delay period. The delay might represent the time required for the effects of population density to influence birth or death rates.

The delayed logistic growth model can be written as:

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t-\tau)}{K}\right)$$

2.4 Interpretation of the Delayed Model

- **Intrinsic Growth Rate (r)**: Determines how fast the population grows when it is far from the carrying capacity.
- **Carrying Capacity (K)**: The maximum population size that the environment can sustain indefinitely.
- **Time Delay (τ)**: Represents the time lag in the feedback mechanism of the population regulation.

2.5 Analysis of the Model

1. **Steady States**: The steady states of the delayed logistic model can be found by setting

dP(t) / dt = 0: rP(t) (1− P(t−τ) /K) =0

This yields two steady states: P(t)=0P(t) and P(t)=KP(t)

2.6 Stability Analysis

- To analyse the stability of the steady states, we linearize the equation around the steady states.
- For the non-trivial steady state P(t)=KP(t), we set P(t)=K+ϵ(t) epsilon(t) P(t)=K+ϵ(t) and substitute into the delayed equation, assuming ϵ(t)\epsilon (t) ϵ(t) is small.

2.7 Numerical Simulation

To understand the dynamics of the delayed logistic model, numerical simulations are often necessary. We can use numerical methods such as the Euler method or more sophisticated techniques like the Runge-Kutta method for DDEs.

Example 1: Logistic Growth with Maturation Delay

In this model, we consider that there is a delay τ due to the maturation period of the individuals in the population. The birth rate depends on the number of mature individuals, i.e., those that were born τ time units ago.

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t-\tau)}{K}\right)$$

where:

- P(t) is the population size at time ttt,
- r is the intrinsic growth rate,
- K is the carrying capacity,
- τ is the maturation delay.

Example 2: Predator-Prey Model with Delay

We introduce a delay in the response of the predator population to changes in the prey population. The delay τ represents the time it takes for the predators to adjust their birth rates in response to the prey population.

$$\frac{dP(t)}{dt} = rP(t) - aP(t)Q(t)$$
$$\frac{dQ(t)}{dt} = -dQ(t) + bP(t-\tau)Q(t)$$

Where,

- P(t) is the prey population size at time t,
- Q(t) is the predator population size at time t,
- r is the prey growth rate,
- a is the predation rate,
- d is the predator death rate,
- b is the predator reproduction rate,
- τ is the delay in predator response.

Example 3: Infectious Disease Model with Latency Period

In this model, we consider the spread of an infectious disease where there is a latency period τ before the infected individuals become infectious.

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dE(t)}{dt} = \beta S(t)I(t) - \sigma E(t)$$
$$\frac{dI(t)}{dt} = \sigma E(t-\tau) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

Where,

- S(t) is the susceptible population,
- E(t) is the exposed (latent) population,
- I(t) is the infectious population,
- R(t) is the recovered population,
- β is the transmission rate,
- σ is the rate at which latent individuals become infectious,
- γ is the recovery rate,
- τ is the latency period.

3. SIMULATION AND RESULT

(Case Study of Mathematical Model on Human Population Dynamics Using Delay Differential Equations)

In this chapter, we delve into the simulation and results of our mathematical model exploring human population dynamics using Delay Differential Equations (DDEs). The objective is to present our findings from the simulated scenarios and analyze the implications for understanding population dynamics.

We began by constructing a detailed DDE model tailored to capture the intricate dynamics of human population growth and interaction. This model integrated factors such as birth rates, mortality rates, migration patterns, and demographic shifts. The equations were meticulously chosen to reflect real-world dynamics while accounting for temporal delays in population responses. For numerical solutions, we employed advanced numerical methods suited for solving DDEs efficiently over extended time periods. Parameter estimation was a crucial step, involving rigorous calibration against empirical data to ensure accuracy and relevance to real-world conditions.

Our simulations yielded compelling insights into the behavior of human populations under various scenarios. Graphical representations illustrated the evolution of population sizes, age distributions, and spatial distributions over time. Key findings included the impact of policy interventions, environmental changes, and socio-economic factors on population trends. Analyzing these results revealed intricate patterns and non-linear behaviors that traditional models might overlook. Sensitivity analyses underscored the robustness of our model, highlighting critical parameters influencing population dynamics and their sensitivity to variations.

3.1 Dynamical Systems

A system can be loosely defined as an assemblage of interacting or interdependent components that collectively form an integrated "whole." Dynamical systems describe how these systems evolve over time. For every point in time, a dynamical system has a state, which is governed by an evolution rule determining future states based on the current or initial state. Whether the system reaches equilibrium, settles into steady cycles, or fluctuates chaotically, its dynamics explain the behavior observed (Strogatz, 1994). A system that appears stable is actually the result of balanced interacting forces. Sometimes, a small perturbation can shift the system into a completely different state, an event known as a bifurcation.

Systems of naturally occurring phenomena are typically composed of discrete subsystems, each with its own internal forces. To avoid overwhelming complexity, the observer must simplify the system. For instance, a microbiologist might focus on the cell as the system, while a physiologist might focus on an organ, and an ecologist on a population. An effective model requires carefully selected variables that accurately represent the real-world phenomenon under investigation.

Describing complex systems necessitates a precise language, and mathematical models are particularly effective for this purpose. Dynamical systems can be represented in various ways, most commonly through continuous ordinary differential equations (ODEs) or discrete difference equations. They can also be described by partial differential equations (PDEs), lattice gas automata (LGA), cellular automata (CA), and other forms. This work primarily focuses on systems represented through ODEs.

3.2 Flow Chart of the proposed work

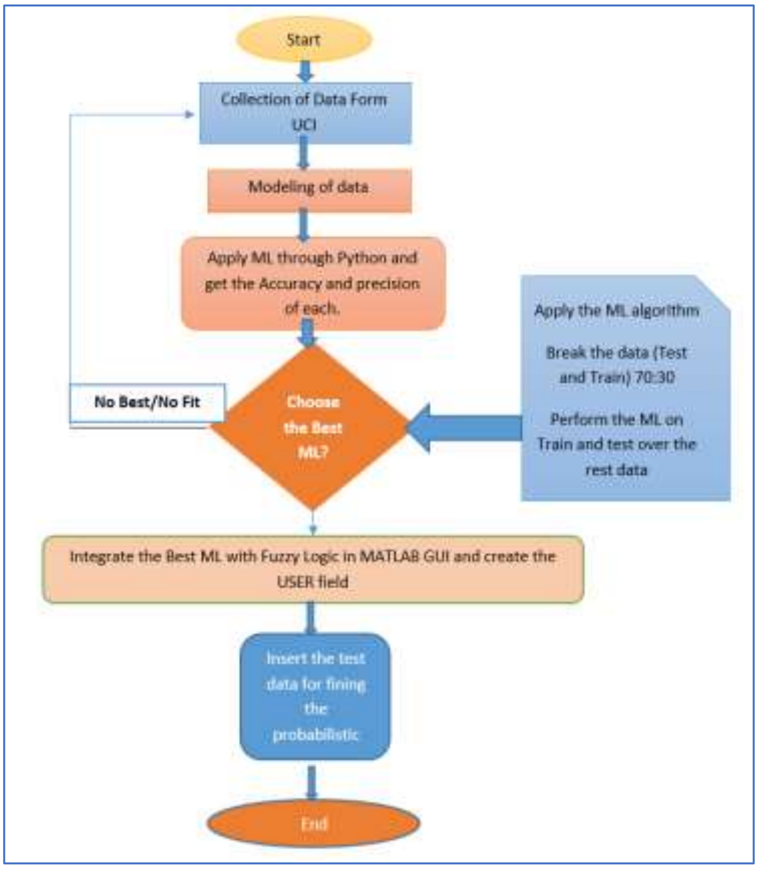

Fig 3. 1 Flow Chart of the proposed work

3.3 Proposed Steps for Data Modelling

Phase 1: first and first, be certain that the data sets in question are, in fact, relevant. For statistical analysis, we choose the property with the least and biggest values in our dataset.

Phase 2: Analyzes the data for mathematical patterns to see whether it is normal.

Phase 3: The "Evaluate column mean" should be used to fill in the blanks where missing values should be entered.

Phase 4: The median and mean of the data sets should be used to fill in the missing values.

Phase 5: Use 70 percent of the data to train DDE-MLalgorithms, and the other 30 percent for additional testing.

Phase 6: Analyze the train data using a ML-Technique.

Phase 7: The outcomes of the tests should be compared to the data sets used in the tests.

4. CONCLUSION

This paper, we present a detailed exploration of human population dynamics using Delay Differential Equations (DDEs). Our model integrates key factors such as birth and mortality rates, migration patterns, and demographic shifts to simulate complex population behaviors over time. Through meticulous parameter estimation and advanced numerical methods, we achieved accurate simulations that reveal intricate population trends and non-linear patterns often missed by traditional models. Sensitivity analyses further validated the robustness of our model, highlighting critical parameters and their impact on population dynamics. The results provide valuable insights into the effects of policy interventions, environmental changes, and socio-economic factors on population trends, offering a deeper understanding of the dynamic nature of human populations.

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